Bayesian Interval Mapping

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1 Overview

Bayesian interval mapping library R/bim provides Bayesian analysis of multiple quantitative trait loci (QTL) models. This includes posterior estimates of the number and location of QTL, and of their effects. This document assumes some familiarity with QTL and with Bayesian methods. In addition it provides graphical diagnostics that can help investigate several 'better' models. Library R/bim requires R/qtl and R/modreg.

```
> library(bim)
```

2 Bayesian Interval Mapping

Consider a simple problem, the 8-week vernalization data for *Brassica napus* used by Satagopan et al (1996).

```
> data(vern)
> summary(vern)

    Backcross

    No. individuals:    104

    No. phenotypes:     1
    Percent phenotyped: 100

    No. chromosomes:    1
        Autosomes:      ch1

    Total markers:      10
    No. markers:        10
    Percent genotyped:  90.9
    Genotypes (%):      AA:41.4  AB:58.6

> plot(vern)
```
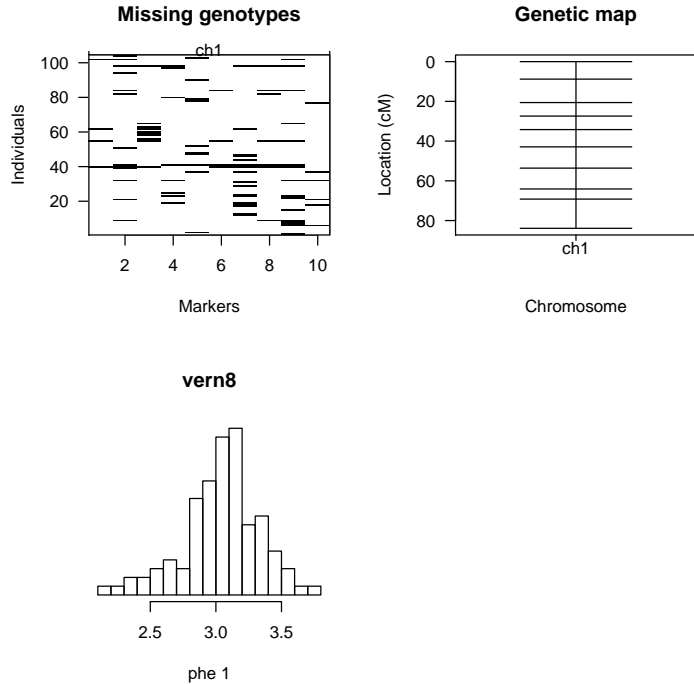
Missing genotypes

Genetic map

vern8

These data are treated as a backcross (although in fact they are from a double haploid, homozygous at every locus). The plot, from library R/qtl, shows the pattern of missing genotypes, the linkage map, and the 11 traits. We focus on `log10flower8`, the logarithm base 10 of the flowering time after eight weeks of vernalization.

Bayesian interval mapping proceeds by first running the `bmapqtl` Markov chain Monte Carlo simulation of the posterior.

```
> bmapqtl.options(prior.nqtl = "poisson")

simulate 400000 MCMC steps, recording by 400 with 0.05 burnin and 0.05 pre-burnin
prior for number of QTL: poisson(3)
initial number of QTL: 0
hyperparameters for priors:
            1  2
init       0.5 -1
prior.mean 1.0 -1
prior.var  3.0 -1
prior.add  0.0  0
prior.dom  0.0  0
random seed: 0

> vernpois.bim = run.bmapqtl(vern)
```

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```
Bayesian interval mapping MCMC run in progress.
Count of 1000 iterations shown separated by dots (negative for burnin):
-20.-15.-10.-5.0.
5.10.15.20.25.30.35.40.45.50.
55.60.65.70.75.80.85.90.95.100.
105.110.115.120.125.130.135.140.145.150.
155.160.165.170.175.180.185.190.195.200.
205.210.215.220.225.230.235.240.245.250.
255.260.265.270.275.280.285.290.295.300.
305.310.315.320.325.330.335.340.345.350.
355.360.365.370.375.380.385.390.395.
```

Now that we have the MCMC simulations, we can examine diagnostic plots. First, time series of the burnin and MCMC runs gives a graphical idea of how well the simulations are 'mixing'.

```
> plot.bim.mcmc(vernpois.bim)
```
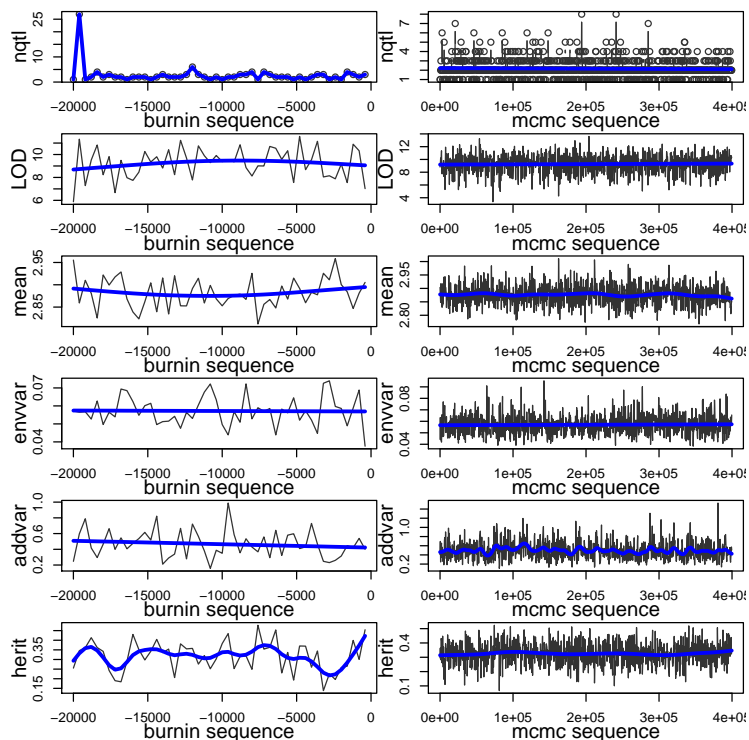

A jittered plot of quantitative trait loci by chromosome shows the where the posterior concentrates along the genome relative to the marker map.

```
> plot.bim.loci(vernpois.bim, vern)
```

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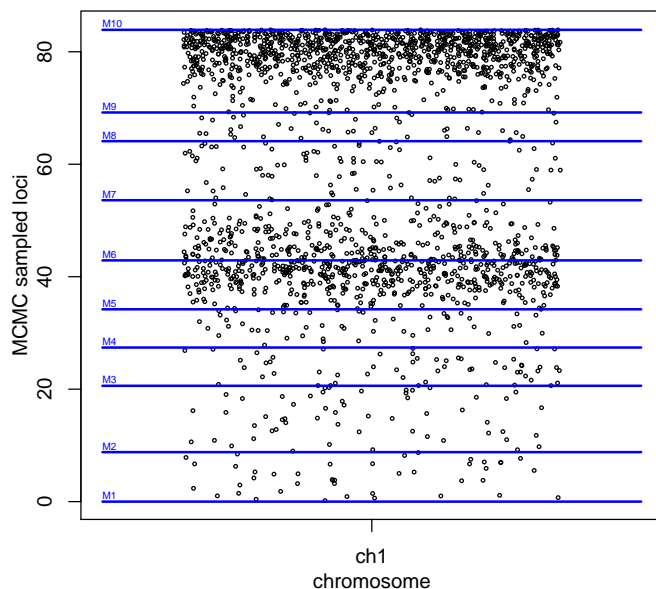
Model selection is assisted by four plots. The top two concern the number of QTL while the bottom two are for the pattern of loci across chromosomes. The left panel shows the posterior as a histogram, overlaid by the prior (rescaled to fit). The right panel show the posterior to prior ratios which make up Bayes factors. A large vertical separation on a log scale indicates substantial difference among models. The summary shows in numbers what is found in the plots.

```
> model = bim.model(vernpois.bim, vern)
> summary(model)

posterior for number of QTL as %
 1  2  3  4  5  6  7  8
22 52 17  6  2  0  0  0
Bayes factor ratios for number of QTL
  1   2   3   4   5   6   7   8
1.0 1.6 0.5 0.2 0.1 0.1 0.1 0.2
model posterior above cutoff 1 as %
pattern
2*1   1 3*1 4*1 5*1
 52  22  17   6   2
Bayes factor ratios for chromosome pattern
pattern
2*1   1 3*1 4*1 5*1
```

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```
1.0 1.3 0.2 0.1 0.0

> plot(model)

$nqtl
$nqtl$posterior

    1     2     3     4     5     6     7     8
0.222 0.519 0.172 0.062 0.016 0.005 0.002 0.002


$nqtl$prior
          1           2           3           4           5           6
0.149361205 0.224041808 0.224041808 0.168031356 0.100818813 0.050409407
          7           8
0.021604031 0.008101512


$nqtl$bf

          1           2           3           4           5           6           7
1.00000000 1.55855856 0.51651652 0.24824825 0.10677344 0.06673340 0.06228451
          8
0.16609202


$nqtl$bfse

          1           2           3           4           5           6           7
0.05919886 0.04744735 0.03583729 0.03053457 0.02647895 0.02976938 0.04399773
          8
0.11732729



$pattern
$pattern$posterior
pattern
  2*1     1   3*1   4*1   5*1
0.519 0.222 0.172 0.062 0.016

$pattern$prior
pattern
       2*1           1         3*1         4*1         5*1
0.14936121 0.04978707 0.22404181 0.22404181 0.16803136

$pattern$bf
pattern
       2*1           1         3*1         4*1         5*1
1.00000000 1.28323699 0.22093770 0.07964033 0.02740313
```

```
$pattern$bfse
pattern
        2*1             1          3*1          4*1          5*1
0.030443099 0.075966162 0.015329245 0.009795772 0.006795754


$param
$param$nqtl
[1] 1

$param$pattern
NULL

$param$exact
[1] FALSE

$param$cutoff
[1] 1


attr(,"class")
[1] "bim.model"
```
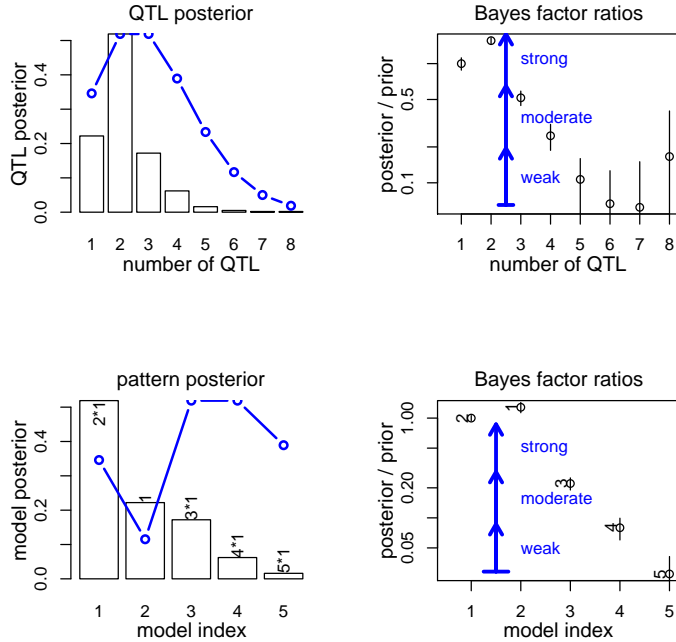
The effects plots show the quantitative trait loci by a histogram and density (blue) on the top plot and effects (additive and possibly dominance) in scatter plot with a smoothing spline fit plus or minus two standard errors. Vertical lines (red) identify the estimated QTL. The summary shows the estimated QTL loci and effects.

```
> qtl = plot.bim.effects(vernpois.bim, vern)
> summary(qtl)

QTL loci and density peaks:
  chr     loci      dens
1 ch1 81.28166 0.0579028

HPD region density cutoffs:
        0.5           0.55          0.6           0.65          0.7           0.75
0.027622297 0.025171976 0.022465807 0.019018241 0.015372926 0.011697632
        0.8           0.85          0.9           0.95
0.007841648 0.004631643 0.003782281 0.002930640

QTL loci and effect estimates:
    chrom     loci       add      add.sd
ch1   ch1 81.28166 0.2217985 0.08198312
     mean        NA 2.8760781 0.03777311
```
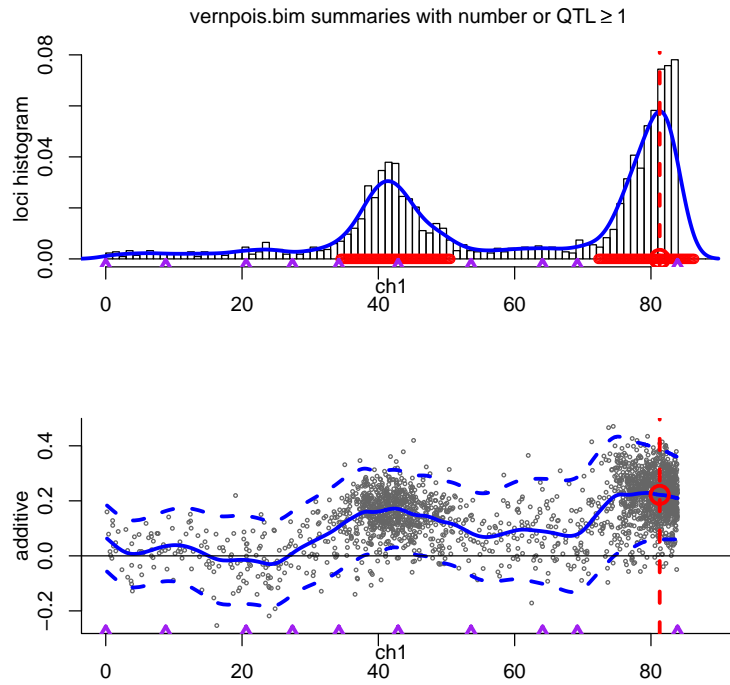
7

```
QTL density estimates by chromosome at 512 grid points with bw = 2

Smoothing spline parameters for additive effects:
      ch1
0.9004665
```


Finally, summary diagnostics for model parameters are shown as histograms and boxplots conditional on the number of QTL. Notice how the boxplots level out as the model gets more complex, although there is very little data for models with a large number of QTL.

```
> plot.bim.diag(vernpois.bim)

LOD 9.372
conditional LOD
      1       2       3       4       5       6       7       8
 7.978   9.633   9.771   9.971   9.699   9.961   9.096  11.138
mean 2.876
conditional mean
     1      2      3      4      5      6      7      8
2.896  2.869  2.873  2.871  2.884  2.870  2.915  2.905
envvar 0.056
```
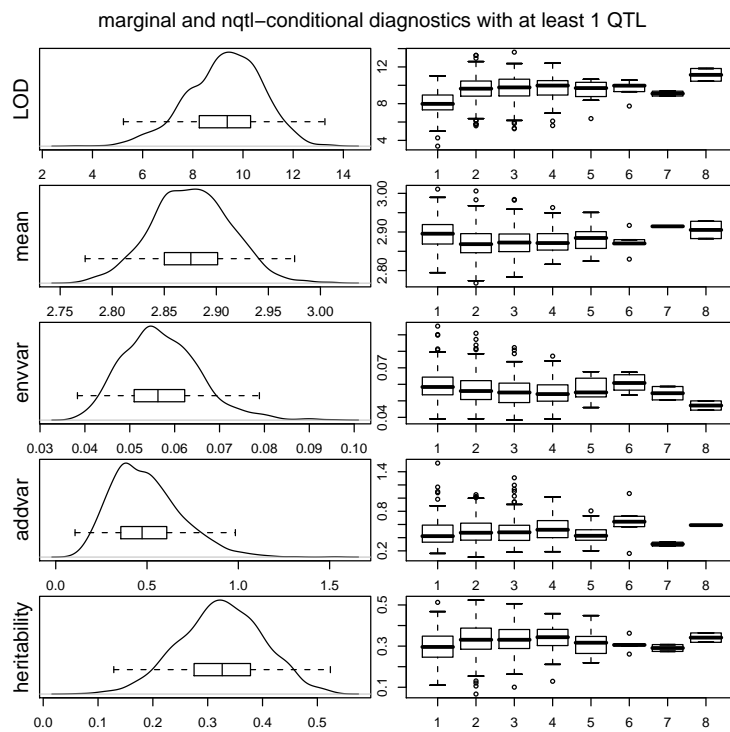
8

```
conditional envvar
    1     2     3     4     5     6     7     8
0.058 0.056 0.055 0.054 0.055 0.061 0.055 0.047
addvar 0.473
conditional addvar
    1     2     3     4     5     6     7     8
0.423 0.475 0.481 0.520 0.429 0.642 0.302 0.590
herit 0.326
conditional heritability
    1     2     3     4     5     6     7     8
0.296 0.331 0.331 0.343 0.316 0.306 0.291 0.341
```

marginal and nqtl–conditional diagnostics with at least 1 QTL

All of these plots can be produced by one call to plot(vernpois.bim,vern). Following this intial investigation, we can refine our graphics by restricting attention to 'better' models. For instance, the model selection suggests two QTL on the chromosome. The option pattern can pick up only those simulations that have 2 QTL on this chromosome. We can then reexamine the model

```
> plot.bim.model(vernpois.bim, vern, pattern = c(1, 1))

$nqtl
$nqtl$posterior
```

```
            2           3           4           5           6           7
0.667095116 0.221079692 0.079691517 0.020565553 0.006426735 0.002570694
            8
0.002570694


$nqtl$prior
            2           3           4           5           6           7
0.224041808 0.224041808 0.168031356 0.100818813 0.050409407 0.021604031
            8
0.008101512


$nqtl$bf

          2          3          4          5          6          7          8
1.00000000 0.33140655 0.15928067 0.06850781 0.04281738 0.03996289 0.10656771


$nqtl$bfse

          2          3          4          5          6          7          8
0.02532657 0.02230198 0.01940591 0.01694993 0.01908689 0.02822169 0.07525783



$pattern
$pattern$posterior
pattern
       2*1        3*1        4*1        5*1
0.66709512 0.22107969 0.07969152 0.02056555


$pattern$prior
pattern
      2*1        3*1        4*1        5*1
0.1493612 0.2240418 0.2240418 0.1680314


$pattern$bf
pattern
       2*1        3*1        4*1        5*1
1.00000000 0.22093770 0.07964033 0.02740313


$pattern$bfse
pattern
        2*1         3*1         4*1         5*1
0.025326572 0.014867985 0.009702953 0.006779970



$param
$param$nqtl
```

```
[1] 1

$param$pattern
[1] 1 1

$param$exact
[1] FALSE

$param$cutoff
[1] 1


attr(,"class")
[1] "bim.model"
```
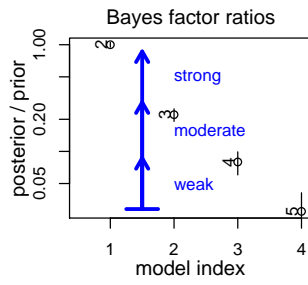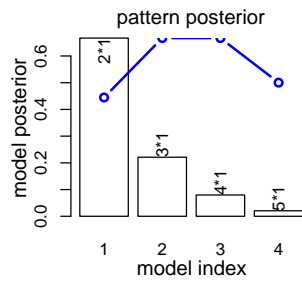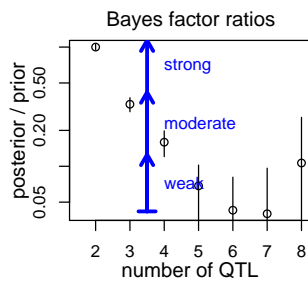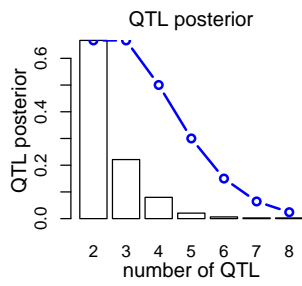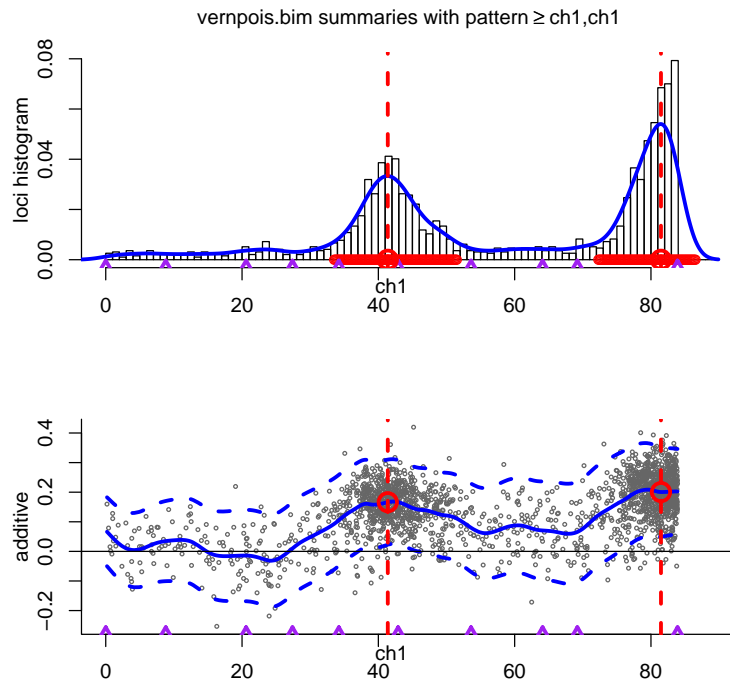

and reexamine the effects

```
> plot.bim.effects(vernpois.bim, vern, pattern = c(1, 1))
```

vernpois.bim summaries with pattern ≥ ch1,ch1

This subsetting is even more effective for full-genome studies. Consider analyzing the full Bnapus dataset for the trait log10flower8.

We can assess the false discovery rate, which gives us some feedback on the width of highest probability density (HPD) regions for QTLs (horizontal red lines on loci histograms).

```
> plot.bim.fdr(vernpois.bim, vern, pattern = c(1, 1))

       H0        M0        M1
0.2353702 0.0000000 1.0000000
$hyp
  H0   M0   M1
0.235 0.000 1.000

$fdr
0.05  0.1 0.15  0.2 0.25
0.19 0.74 0.87 0.95 0.99
```